BAYESIAN ESTIMATION OF STOCHASTIC-VOLATILITY JUMP-DIFFUSION MODELS ON INTRADAY PRICE RETURNS

MILAN FÍČURA
University of Economics, Faculty of Finance and Accounting, Department of Banking and Insurance, Winston Churchill Square, 130 67, Prague 3, Czech Republic
email: milan.ficura@vse.cz

JIŘÍ WITZANY
University of Economics, Faculty of Finance and Accounting, Department of Banking and Insurance, Winston Churchill Square, 130 67, Prague 3, Czech Republic
email: jiri.witzany@vse.cz

Abstract
Methodology is presented of how to apply Stochastic-Volatility Jump-Diffusion (SVJD) models with self-exciting jumps to the intraday asset price returns (specifically to the EUR/USD 4-hour returns). On the intraday frequencies it is necessary to cope with the intraday seasonality of volatility and jump intensity. This is achieved in the new model by using a simple multiplicative structure. A MCMC algorithm is presented for the estimation of model parameters, seasonality adjustments and latent state variables. It is shown that the estimated stochastic volatilities and jumps using the intraday SVJD model correspond more closely to their non-parametric estimates, constructed using power-variation estimators, than when a daily SVJD model is used. Most importantly, the number of identified jumps increases by an order of magnitude in the intraday SVJD model compared to the daily one. As an additional result it is found that the jumps with high Bayesian probability of occurrence do exhibit clustering while the ones with lower probabilities of occurrence do not.

Key words: Stochastic volatility, Self-exciting jumps, Bayesian inference.

1. Introduction

The ability to accurately model and forecast volatility and jumps in the financial time series plays a crucial role in a wide area of financial applications including option pricing, VaR estimation, optimal portfolio construction and quantitative trading. In the last decade the ability of financial econometricians to identify and forecast volatility and jumps greatly increased due to the increased availability of intraday price returns. The commonly used approach utilizes the asymptotic theory of power variations (Andersen et al., 2007) in order to non-parametrically estimate the quadratic variation, integrated variance and jump variance of the price process, which can then be modelled by standard time series models.

This approach proved to be highly successful particularly in the area of volatility forecasting, and as numerous studies have shown, models such as ARFIMA-RV applied to realized variance calculated from high-frequency returns easily beat traditional volatility forecasting models such as EWMA and GARCH, as well as the more complex Stochastic-Volatility Jump-Diffusion (SVJD) models applied to daily data.

Nevertheless the use of realized variance and other aggregated measures in volatility forecasting and jump identification is associated with many difficulties as the unbiasedness and robustness of these estimators lay on unrealistic assumptions that hold only
asymptotically. In real-life applications where the returns sampling frequency is always finite and the high-frequency returns are plagued by microstructure noise and intraday seasonality effects, the aggregated volatility estimators may be biased and the bias is even more pronounced for the estimators of price jumps. Therefore we offer here an alternative approach of how to utilize the intraday returns for volatility and jump modeling by incorporating these returns into the Stochastic-Volatility Jump-Diffusion (SVJD) model setting.

The main theoretical edge of the SVJD models is that they are trying to model the real underlying dynamics of the latent volatility and jumps instead of the dynamics of some noisy estimators as is the cases of the other classes of models. Furthermore the rapid increase in computation power in the last decade as well as the new developments in Bayesian estimation methods such as MCMC techniques, EMM methods and particle filters rendered the traditionally problematic estimation of these models far more tractable.

One possible approach of how to utilize the intraday returns in the SVJD model setting is to use the non-parametric power-variation estimators in the MCMC estimation of these models (Takahashi et al., 2009). Although promising, this approach is still dependent on the quality of the non-parametric estimates and is thus susceptible to the already mentioned biases due to microstructure noise and intraday seasonality effects.

Therefore we will pursue a different approach in this paper, in which the SVJD models are applied directly to the intraday price returns, which was rarely pursued in past due to estimation difficulties. In the SVJD models, every time-step corresponds to one or more latent state variables (stochastic volatilities, jump occurrences, jump sizes, etc.) which have to be estimated. With the increase of the return frequency from the daily to the intraday ones, the number of latent states increases dramatically, typically from several thousand to tens of thousands, thus increasing the computational demand of the estimation significantly.

In addition to that, the volatility and jumps behave differently on the intraday frequencies than on the daily frequency with the most important issue being the intraday seasonality of volatility and jump intensity which has to be incorporated into the model.

One of the few studies in which SVJD models were applied directly to the intraday returns was performed by Stroud and Johannes (2014). The authors used a complex two-factor stochastic volatility model with jumps in returns and volatility to fit the S&P 500 futures volatility in-sample (using MCMC algorithm) as well as for out-of-sample predictions (using particle filters). The results of the study are highly promising as the out-of-sample forecasts dominated all of the benchmark models in several different applications including volatility forecasting, VaR estimation and volatility trading.

In comparison to the mentioned study our model uses just a single-factor stochastic volatility model and it lacks the jumps in volatility. It does however use a more advanced approach for the modelling of the price jump component, by utilizing the self-exciting Hawkes process instead of the i.i.d Poisson process, thus allowing for the empirically observed jump clustering effects. These may possibly play an important role in areas such as VaR estimation and out-of-the-money option pricing, see (Fulop, Li and Yu, 2014).

The model is applied to the EUR/USD time series, specifically to its 4-hour logarithmic returns. The rest of the paper is organized as follows. In chapter 2 the intraday SVJD model is presented. In chapter 3 the MCMC estimation procedure is explained and in chapter 4 the empirical application of the model to the EUR/USD time series is performed. In the conclusion we sum up the main results and propose areas for futures research.
2. The intraday SVJD model

Although SVJD models are traditionally specified in continuous-time, in this paper a discretized version of the model is presented. The discretization was performed by using the Euler method with a time step \( t \) equal to a 4-hour period. So the logarithmic returns follow:

\[
r(t) = \mu + \sigma(t)\varepsilon(t) + J(t)Q(t)
\]

(1)

Where \( r(t) \) is the 4-hour logarithmic return defined as \( r(t) = p(t) - p(t-1) \), and \( p(t) \) is the logarithm of the closing price for the given period. Parameter \( \mu \) represents the mean one-period return, \( \sigma(t) \) is the stochastic volatility, \( \varepsilon(t) \sim N(0,1) \) is a standard normal white noise, \( J(t) \sim N(\mu_j, \sigma_j) \) is a variable determining the size of the jumps and \( Q(t) \sim Bern[\lambda(t)] \) is the jump occurrence indicator following a Bernoulli process with intensity \( \lambda(t) \).

The stochastic volatility \( \sigma(t) \) is further decomposed into its persistent and seasonality components using a simple multiplicative structure:

\[
\sigma(t) = v(t)s(t)
\]

(2)

Where \( v(t) \) denotes the persistent component of the stochastic volatility and \( s(t) \) its seasonal component. The persistent component \( v(t) \) is modelled using the log-variance model. Thus, denoting \( V(t) = v^2(t) \), the variable \( h(t) = \log(V(t)) \) follows an AR(1) process:

\[
h(t) = \alpha + \beta h(t-1) + \gamma \varepsilon_v(t)
\]

(3)

Where \( \alpha = (1-\beta)\theta \) is the constant, \( \theta \) is the long-term mean, \( \beta \) is the autoregressive coefficient, \( \gamma \) is the volatility of the volatility and \( \varepsilon_v(t) \sim N(0,1) \) is the white noise of the volatility process, which is in our case (the currency market) uncorrelated with \( \varepsilon(t) \).

The seasonal volatility component \( s(t) \) is modeled deterministically as follows:

\[
s(t) = \sum_{j=1}^{6} s_j d_j(t)
\]

(4)

Where \( s_j \) is a parameter representing the multiplicative adjustment of the log-variance at season \( j \) and \( d_j(t) \) is a dummy variable equal to one at season \( j \) and zero otherwise. The summation goes from 1 to 6, as they are six 4-hour seasons in a 24-hour day. Consequently 5 dummy parameters have to be estimated as one is the benchmark automatically set to one.

Finally it is necessary to define the model of jump intensity \( \Pr[Q(t) = 1] = \lambda(t) \). Similarly as in the case of volatility a multiplicative structure is assumed:

\[
\lambda(t) = \lambda_H(t)\lambda_S(t)
\]

(5)

Where \( \lambda_H(t) \) denotes the self-exciting component following a discretized Hawkes process and \( \lambda_S(t) \) denotes the seasonal component which is deterministic.

The Hawkes process of the self-exciting component looks as follows:

\[
\lambda_H(t) = \alpha_H + \beta_H \lambda_H(t-1) + \gamma_H Q(t-1)
\]

(6)
Where \( \alpha_j = (1 - \beta_j - \gamma_j) \theta_j \) and \( \theta_j \) is the long-term jump intensity, \( \beta_j \) is the decay rate parameter and \( \gamma_j \) is the self-exciting parameter. So we can see the nature of the Hawkes process in which the jump intensity increases after every jump occurrence initially by \( \gamma_j \), but as time passes it the decays back exponentially, by the rate \( \beta_j \), to its long-term level \( \theta_j \).

The seasonal component of jump intensity \( \lambda_s(t) \) is modelled as:

\[
\lambda_s(t) = \sum_{j=1}^{6} \lambda_{s,j} d_j(t)
\]

(7)

Where \( \lambda_{s,j} \) represents the multiplicative intensity adjustment corresponding to the season \( j \) and \( d_j(t) \) is a dummy variable equal to one at season \( j \) and zero otherwise.

Altogether the intraday SVJD model has 9 parameters related to the dynamics of the stochastic processes, namely: \( \mu, \alpha, \beta, \gamma, \theta_1, \beta_1, \gamma_1, \mu_1, \sigma_1 \) and 10 additional parameters associated with the intraday seasonality effects: \( s_j \) and \( \lambda_{s,j} \), with \( j \) going from 1 to 5 (as one of the 6 seasons is chosen to be benchmark with parameter value equal to one). In addition to that, 3 vectors of latent state variables have to be estimated, \( V, J, Q \), corresponding to the stochastic variances, jump sizes and jump occurrences. The latent jump intensities do not have to be estimated as they are deterministic given the jump occurrences and the parameters.

3. MCMC estimation

The parameters and latent state variables of the SVJD model are estimated using a MCMC (Markov Chain Monte Carlo) algorithm based on Witzany (2013), Jacquier et al. (2007) and Johannes and Polson (2009).

MCMC is a Bayesian estimation method, used for the sampling from high-dimensional posterior densities of model parameters by constructing a Markov Chain that uses only the information about the shapes of the univariate conditional densities of the model parameters.

A well-known MCMC technique is the Gibbs sampler. Let denote the parameter vector as \( \Theta = (\theta_1, ..., \theta_k) \) and its Bayesian joint posterior density as \( p(\Theta \mid \text{data}) \). As it is often impossible to analytically express the joint density or sample from it, the Gibbs sampler allows us to construct a Markov Chain by using the information about the conditional densities \( p(\theta_j \mid \theta_i, i \neq j, \text{data}) \) which are far easier to express and sample from, and the constructed chain should asymptotically converge to the joint density \( p(\Theta \mid \text{data}) \).

The procedure of Gibbs Sampler is as follows:

0. Assign a vector of initial values to \( \theta^0 = (\theta_1^0, ..., \theta_k^0) \) and \( j = 0 \)
1. Set \( j = j + 1 \)
2. Sample \( \theta_i^j \mid p(\theta_1^j, ..., \theta_{i-1}^j, \text{data}) \)
3. Sample \( \theta_i^j \mid p(\theta_1^j, \theta_2^j, ..., \theta_{i-1}^j, \text{data}) \)

... k+1. Sample \( \theta_k^j \mid p(\theta_1^j, \theta_2^j, ..., \theta_{k-1}^j, \text{data}) \) and return to step 1.

The conditional densities \( p(\theta_j \mid \theta_i, i \neq j, \text{data}) \) are usually obtained by applying the Bayes theorem to the likelihood function and the prior density as follows:
\[ p(\theta_1^j, \theta_2^{j-1}, \ldots, \theta_k^{j-1}, \text{data}) \propto L(\text{data} | \theta_1^j, \theta_2^{j-1}, \ldots, \theta_k^{j-1}) \ast \text{prior}(\theta_1^j, \theta_2^{j-1}, \ldots, \theta_k^{j-1}) \]  

(8)

Where \( L() \) denotes the likelihood function, \( \text{prior}() \) the Bayesian prior density and \( \propto \) is a proportionate relationship. In order to use the Gibbs sampler it is necessary to normalize the right hand side of equation 8 (i.e. replace the proportionate with equality) by dividing it by its integral over \( \theta_1 \), which represents the density \( p(\text{data} | \theta_2^{j-1}, \ldots, \theta_k^{j-1}) \).

The calculation of the integral is often impossible. In these cases another MCMC technique may be used called the Metropolis-Hastings algorithm.

Metropolis-Hastings algorithm is a rejection sampling algorithm which draws in every step a proposal value of the given parameter from a proposal density \( q \) and then either accepts it or declines it based on a given probability. Specifically, Step 2 in the Gibbs Sampler is replaced by the following two step procedure:

A. Draw \( \theta_1^j \) from the proposal density \( q(\theta_1^j | \theta_2^{j-1}, \ldots, \theta_k^{j-1}, \text{data}) \)

B. Accept \( \theta_1^j \) with a probability \( \alpha = \min(R,1) \), where \( R \) denotes the acceptance ratio:

\[ R = \frac{p(\theta_1^j, \theta_2^{j-1}, \ldots, \theta_k^{j-1}, \text{data} | q(\theta_1^j | \theta_2^{j-1}, \ldots, \theta_k^{j-1}, \text{data}))}{p(\theta_1^{j-1}, \theta_2^{j-1}, \ldots, \theta_k^{j-1}, \text{data} | q(\theta_1^{j-1} | \theta_2^{j-1}, \ldots, \theta_k^{j-1}, \text{data}))} \]  

(9)

There are several different versions of the Metropolis-Hastings algorithm differing in the choice of the proposal densities. One possible version is the Random-Walk Metropolis-Hastings with the proposal density given as:

\[ \theta_1^j \sim \theta_1^{j-1} + N(0,c) \]  

(10)

Where \( c \) is a meta-parameter, influencing the computational efficiency of the algorithm but not its asymptotical properties.

A great advantage of the Random-Walk Metropolis-Hastings algorithm is that its proposal distribution is symmetric. By utilizing the relationship 8 we can see that the acceptance ratio 9 reduces to the likelihood ratio:

\[ R = \frac{L(\text{data} | \theta_1^j, \theta_2^{j-1}, \ldots, \theta_k^{j-1})}{L(\text{data} | \theta_1^{j-1}, \theta_2^{j-1}, \ldots, \theta_k^{j-1})} \]  

(11)

Another popular sampling algorithm is the Independence Sampling Metropolis-Hastings in which the proposal density does not depend on the current value of the given parameter. In this case the proposal densities in the acceptance ratio do not cancel out. Furthermore, in order to use the algorithm efficiently, it is necessary to choose the proposal density so that it matches closely the shape of the target density.

In our case we want to estimate a vector of a few model parameters and a large number of latent state variables. Both of these will be estimated iteratively using MCMC algorithm combining Gibbs sampler, Random-Walk Metropolis-Hastings and Independence Sampling.

The estimation algorithm proceeds as follows (for \( g = 1, \ldots, N \)):

1. Sample reasonable initial values \( \mu^{(0)}, \lambda^{(0)}, \sigma_j^{(0)}, \alpha^{(0)}, \beta_j^{(0)}, \gamma^{(0)}, \phi, \nu^{(0)}, J^{(0)}, Q^{(0)} \)

2. For \( i = 1, \ldots, T \) sample new jump sizes using relationships:

\[ J_i \sim \phi(J; \mu_j^{(i-1)}, \sigma_j^{(i-1)}) \text{ if } Q_i^{(i-1)} = 0 \]
3. For $i = 1, ..., T$ sample new jump occurrences: $Q_i^\varepsilon \in \{0, 1\}$ with probabilities

$$P_i[Q = 1] = p_i (p_0 + p_i),$$

where $p_0 = \phi(r_i^\varepsilon, \mu, \sigma_i^\varepsilon, \nu)$ and $p_i = \phi(r_i^\varepsilon, \mu, \sigma_i^\varepsilon + J_i, \nu)$.  

4. Sample new volatility seasonality adjustments $s_i^\varepsilon$ using normally distributed time series $s_i^\varepsilon = (r_i - \mu - J_i^\varepsilon Q_i^\varepsilon) / \sqrt{V_i^\varepsilon}$, as for $d_{ij} = 1$ holds that $s_i^\varepsilon \sim N(0, s_j^\varepsilon)$ and we can sample $s_i^\varepsilon \sim IG \left( \frac{1}{2} \sum_{i=1}^{T} \frac{1}{2} \left( \sum_{i=1}^{T} d_{ij} \right) \right)$.  

5. Sample new stochastic variances $V_i^\varepsilon$ for $i = 1, ..., T$ using Independence Sampling Metropolis-Hasting routine proposal density based on Jacquier et al. (1994):

$$g(V_i | V_{i-1}, \Theta, r, J, Q_j) = IG(V_i; \phi + 0.5, \phi - 1) \exp(\mu_i + 0.5\sigma_i^2 V_i) \exp(0.5(r_i - J_i, Q_j, S_i^2 V_i^\varepsilon))$$

where $\phi = 1 - 2\exp(\gamma^2) / \exp(\gamma^2)$, $\mu_i = (a(1 - \beta) + h_i \log(V_i)) / (1 + \beta^2)$ and $\gamma = \sqrt{1 + \beta^2}$.  

6. Sample new stochastic volatility autoregression coefficients $\alpha_i^\varepsilon, \beta_i^\varepsilon, \gamma_i^\varepsilon$ from $h_i = \log(V_i^\varepsilon)$ for $i = 1, ..., T$ using Bayesian linear regression model (Lynch, 2007) $\beta = (X'X)^{-1} X y$, $e = y - X \beta$, $X = \left(1 \ldots 1\right)$ and $y = (h_2 \ldots h_T)$, so we sample:

$$(\alpha_i^\varepsilon, \beta_i^\varepsilon, \gamma_i^\varepsilon) \sim IG \left( \frac{n - 2}{2}, \frac{e'e}{2} \right)$$

7. Sample $\mu_i^\varepsilon$ based on the normally distributed time series $r_i - J_i^\varepsilon Q_i^\varepsilon$ with variances $V_i^\varepsilon S_i^\varepsilon$ (denoting $S_i^\varepsilon = (s_i^\varepsilon)^2$):

$$p(\mu_i^\varepsilon | r_i^\varepsilon, Q_i^\varepsilon, V_i^\varepsilon, S_i^\varepsilon) \propto \left( \mu_i^\varepsilon \sum_{i=1}^{T} \frac{r_i - J_i^\varepsilon Q_i^\varepsilon / \sqrt{V_i^\varepsilon S_i^\varepsilon}}{\sum_{i=1}^{T} \frac{1}{\sqrt{V_i^\varepsilon S_i^\varepsilon}}} \sum_{i=1}^{T} \frac{1}{\sqrt{V_i^\varepsilon S_i^\varepsilon}} \right)$$

8. Sample new Hawkes process parameters $\Theta_j, \beta_j, \gamma_j$ and all of the seasonality adjustments $\lambda_{s,j}$ using Random-Walk Metropolis-Hasting routine proposal density $\theta_j^\varepsilon = \theta_j^{\varepsilon-1} + N(0, \sigma_j^\varepsilon)$ (or analogically for $\beta_j, \gamma_j, \lambda_s$) and the likelihood function:

$$L(Q_j^\varepsilon | \theta_j, \beta_j, \gamma_j, \lambda_s) = \prod_{i=1}^{T} f(t_i, \lambda_{s,j}^0 (1 - \lambda_j)^{-0})$$

9. Sample $\mu_j^\varepsilon, \sigma_j^\varepsilon$ based on the normally distributed time series $J_i^\varepsilon$ and uninformative priors $p(\mu) \propto 1$ and $p(\log \sigma)^2 \propto \sigma$ (equivalent to $p(\sigma^2) \propto \sigma / \sigma^2$):

$$p(\mu_j^\varepsilon | J_i^\varepsilon, \sigma_j^\varepsilon) \propto \left( \sum_{i=1}^{T} \frac{J_i^\varepsilon}{T} \right), p(\sigma_j^\varepsilon | J_i^\varepsilon, \mu_j^\varepsilon) \propto IG \left[ \sigma_j^\varepsilon, \frac{T}{2} \sum_{i=1}^{T} (J_i^\varepsilon - \mu_j^\varepsilon)^2 \right]$$
4. Empirical application

The intraday SVJD model was applied to the time series of the EUR/USD exchange rate 4-hour returns in the period between 1.11.1999 and 10.10.2014, containing altogether 23337 observations. The data were provided by ForexHistoryDatabase.com and the time labeling corresponds to the time-zone UTC+2.

The MCMC estimation algorithm was implemented in Matlab, using a chain with 3000 iterations out of which the first 2000 were discarded and the remaining 1000 were used for the parameter estimation based on the posterior means. During the 3000 iterations most of the parameters converged quickly to their posterior densities with the exception of the betaJ parameter whose convergence was problematic and will be discussed later in more detail. For the estimation of the seasonality the period between 8:00 and 12:00 was chosen as benchmark (i.e. the multiplicative adjustments for this period were set to 1).

In addition to the intraday SVJD model, the daily model (i.e. using daily returns and no seasonality) was estimated as well, and it will be sometimes used for comparisons.

Table 1 shows the parameter estimates of the intraday SVJD model based on the MCMC posterior means and the Bayesian estimates of their standard errors.

Table 1. Bayesian mean parameter estimates and the Bayesian standard errors

<table>
<thead>
<tr>
<th>m</th>
<th>mJ</th>
<th>sigmaJ</th>
<th>alpha</th>
<th>beta</th>
<th>gamma</th>
<th>thetaJ</th>
<th>betaJ</th>
<th>gammaJ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>4E-05</td>
<td>-7E-05</td>
<td>0.0037</td>
<td>0.9975</td>
<td>0.0566</td>
<td>0.1050</td>
<td>0.3638</td>
<td>0.0070</td>
</tr>
<tr>
<td>Std</td>
<td>9E-06</td>
<td>1E-04</td>
<td>0.0001</td>
<td>0.0096</td>
<td>0.0008</td>
<td>0.0064</td>
<td>0.0081</td>
<td>0.2695</td>
</tr>
</tbody>
</table>

We can see from Table 1 that the beta parameter describing the persistence of the autoregressive volatility component is close to one. Its value is however still more than three standard deviations away from 1, rendering the AR(1) process of the log-variance stationary, although highly persistent. This is in accordance with the empirically observed long-memory of volatility. The daily SVJD model achieved a beta equal to 0.9959.

Concerning the jump component, the mean jump size (mJ) is slightly negative and the standard deviation of the jump size (sigmaJ), corresponding the mean jump magnitude, is relatively low (only 0.37%) and significantly lower than in the case of the daily model (which is 0.78%). The results of the intraday SVJD model are thus (in this regard) more similar to the results of the non-parametric methods of jump estimation (finding a lot of small jumps) then to the results of the daily SVJD model, see (Fičura and Witzany, 2014).

From the Hawkes process parameter estimates there does not seem to be any jump clustering in the time series, as the mean estimate of betaJ is very low and has a large standard error. Later in the text we will show that the Bayesian distribution of betaJ is actually bimodal with additional (lower) mode at value of around 0.99 (indicating strong clustering effects).

In addition to the parameters above, 10 seasonality adjustment parameters have been estimated, 5 corresponding to the stochastic volatility and 5 corresponding to the jump intensity. As already mentioned, the value of the volatility and jump intensity in the 4-hour period between 8:00-12:00 AM was chosen as benchmark with parameter values set to one. The other parameters represent multiplicative adjustments of the volatility (in the form of standard deviation) and jump intensity compared to the benchmark period. The values of the parameter estimates and their Bayesian standard errors are shown in Table 2.
Table 2. Intraday seasonality adjustments for volatility and jump intensity

<table>
<thead>
<tr>
<th>Time period</th>
<th>0:00-4:00</th>
<th>4:00-8:00</th>
<th>8:00-12:00</th>
<th>12:00-16:00</th>
<th>16:00-20:00</th>
<th>20:00-24:00</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Volatility adj.</strong> Mean</td>
<td>0.5815</td>
<td>0.4817</td>
<td>1.0000</td>
<td>0.9239</td>
<td>0.8963</td>
<td>0.5429</td>
</tr>
<tr>
<td><strong>Std</strong></td>
<td>0.0117</td>
<td>0.0074</td>
<td>0.0000</td>
<td>0.0290</td>
<td>0.0225</td>
<td>0.0088</td>
</tr>
<tr>
<td><strong>Intensity adj.</strong> Mean</td>
<td>1.0734</td>
<td>0.3074</td>
<td>1.0000</td>
<td>2.5996</td>
<td>2.4130</td>
<td>0.7857</td>
</tr>
<tr>
<td><strong>Std</strong></td>
<td>0.1184</td>
<td>0.0522</td>
<td>0.0000</td>
<td>0.3414</td>
<td>0.2480</td>
<td>0.0957</td>
</tr>
<tr>
<td><strong>Intensity (long-term)</strong></td>
<td>0.1127</td>
<td>0.0323</td>
<td>0.1050</td>
<td>0.2729</td>
<td>0.2533</td>
<td>0.0825</td>
</tr>
</tbody>
</table>

Source: Authorial computation

As can be seen from Table 2, the stochastic volatility is highest in the morning period (i.e. 8:00-12:00) and it is only slightly lower (92% of the morning volatility) in the afternoon period (12:00-16:00) and in the evening period (16:00-20:00) (90%). In the remaining seasons the volatility is significantly lower, equal to approximately 50%-60% of the volatility in the morning period. The lowest volatility corresponds to the period between 4:00 and 8:00 AM.

The seasonality of the jump intensity follows a slightly different pattern. Most importantly, the jump intensity is not highest during the morning period (8:00-12:00), but in the afternoon and evening periods (12:00-16:00 and 16:00-20:00) when it is about two and half times as big as in the morning period. Considering the long-term jump intensity in the morning period (10.5%) the multiplicative adjustments result in approximately 27% and 25% average jump intensities in the afternoon and evening periods, indicating that the jumps occur very frequently in these periods. The reason for this might be the timing of the US economic news announcements, but this hypothesis would have to be further examined.

Together with the parameters and seasonality adjustments, 3 vectors of latent state variables have been estimated, corresponding to the stochastic variances, jump occurrences and jump sizes. Figure 1 (left) shows the mean posterior estimates of the stochastic variances.

![Figure 1. Bayesian estimates of the stochastic variances and jump occurrences](image)

Source: Authorial computation

From the volatility estimates in Figure 1 (left) there can be seen the effect of the global financial crisis (after the 14000th period) during which the variances increased dramatically.

The daily volatility estimates of the intraday SVJD model and the daily SVJD model were compared in their ability to fit the non-parametric estimates of the daily integrated variance constructed using the methodology developed in Andersen et al. (2007). The results of the Minzer-Zarnowitz regressions showed that the intraday SVJD model achieved a smaller bias and higher R-squared (0.5505) compared to the daily model (0.4849).
Figure 1 (right) shows the estimates of the Bayesian probabilities of jump occurrences. It is apparent that the intraday SVJD model identified a relatively large number of jumps in the EUR/USD time series. Indeed, it found 318 jumps with Bayesian probability of occurrence larger than 95%. The daily SVJD model, on the other hand, identified only 2 jumps with Bayesian probability of occurrence larger than 95%.

We decided to compare to what degree do the jumps identified by the intraday and the daily SVJD models correspond to the jumps identified non-parametrically using the Z statistics described in Andersen et al. (2007). The Spearman rank correlation of Bayesian jump probabilities of the daily SVJD model with the Z statistics is only 0.0255, while for the intraday SJVD model it is 0.1332, indicating a closer correspondence.

Finally, let’s return to the analysis of the betaJ parameter convergence. As already mentioned, the betaJ parameter did not converge very well and its estimated posterior marginal distribution is bimodal. The bimodality is even more pronounced in the posterior marginal bivariate distribution of betaJ and gammaJ. The shape of this distribution (estimated using bivariate kernel density) can be seen on Figure 2.

Figure 2. Posterior marginal bivariate density of the betaJ and gammaJ parameters

Source: Authorial computation

In order to further examine the issue, the Hawkes process was re-estimated for several different jump-time-series identified with a given Bayesian probabilities of occurrence or higher based on the intraday SVJD model. The results are in Table 3.

Table 3. Hawkes process parameters estimated for the previously identified jumps on different probability levels (based on their Bayesian probabilities of occurrence)

<table>
<thead>
<tr>
<th>prob.</th>
<th>count</th>
<th>thetaJ</th>
<th>betaJ</th>
<th>gammaJ</th>
</tr>
</thead>
<tbody>
<tr>
<td>95%</td>
<td>318</td>
<td>0.0016</td>
<td>0.9930</td>
<td>0.0010</td>
</tr>
<tr>
<td>75%</td>
<td>624</td>
<td>0.0040</td>
<td>0.9928</td>
<td>0.0013</td>
</tr>
<tr>
<td>50%</td>
<td>1173</td>
<td>0.0099</td>
<td>0.9921</td>
<td>0.0016</td>
</tr>
<tr>
<td>25%</td>
<td>3231</td>
<td>0.0516</td>
<td>0.1841</td>
<td>0.0033</td>
</tr>
</tbody>
</table>

Source: Authorial computation

From Table 3 it is apparent that the jumps with probability of occurrence larger than 50% do exhibit some clustering effects as the betaJ parameters are close to 1. The jumps with lower probabilities of occurrence do however not exhibit clustering at all. This phenomenon probably caused the bimodality in the parameter estimation as mentioned above.
5. Conclusion

A methodology was presented of how to cope with the intraday seasonality of volatility and jump intensity in the SVJD models when applied to high-frequency returns. The developed MCMC estimation procedure exhibited good convergence for most of the parameters with the exception of the decay parameter of the jump clustering effects (betaJ) whose posterior distribution was found to be bimodal.

Considering the estimated stochastic volatilities and jumps, the results of the intraday SVJD model corresponded more closely (compared to the daily SVJD model) to the non-parametrically estimated volatilities and jumps. Most importantly the intraday SVJD model identified significantly more jumps in the time series than the daily SVJD model.

In the future research we plan to further examine the clustering behavior of the jump component. Another extension of the model would be to add jumps in the volatility process and to model the stochastic volatility by using multiple components with different levels of persistence. Finally, for practical applications it will be useful to extend the methodology in order to enable out-of-sample forecasts (for example through particle filters).

Acknowledgements

The support of the grant scheme “Advanced methods of financial asset returns and risks modelling” IGA VŠE F1/23/2015 is gladly acknowledged.

References