18th AMSE Applications of Mathematics and Statistics in Economics
International Scientific Conference | Jindřichův Hradec, 2-6 September 2015

CORPORATE FAILURE PREDICTION USING DEA: AN APPLICATION TO COMPANIES IN THE SLOVAK REPUBLIC

VIERA ROHÁČOVÁ, PAVOL KRÁĽ
Matej Bel University BanskáBystrica, Faculty of Economics, Department of Quantitative Methods and Information Systems, Tajovského 10, 975 90 BanskáBystrica, Slovakia
email:viera.rohacova@umb.sk, pavol.kral@umb.sk

Abstract
In the paper we discuss corporate failure prediction based on data envelopment analysis (DEA) technique. The main aim of the paper is to develop a simple bankruptcy assessment tool providing an information basis for financial decisions of companies in order to prevent possible losses.Using a specific input-output selection procedure we identify the Corporate Failure Frontier (CFF) that indicates firms that are about to fail. The proposed methodology uses financial ratios to predict financial distress. Since the financial ratios can take in many casesnegative values, the modification of the additive model is used. Using theproposed two-step procedure,we classify firms under consideration into three zones: distress zone, grey zone and safe zone.Some properties of the proposed methodology are illustrated on a sample of Slovak companies.

Key words: *Data envelopment analysis, corporate failure prediction.*

1. Introduction

Corporate failure may have various forms, numerous manifestations and consequences. Sincecorporate failure imposes significant costs on firm´s stakeholders (see e.g. (Warner, 1977)), early identification of the risk of corporate failure isan important issue studied in both theoretical and practical sphere of corporate governance. Developing and verifyingthe most reliable techniques to identify threatsof the corporate failure areimportantforthe following reasons. First, the corporate leaders may adjusttheir decisionsaccording to emergingadversesituationandthusavertheimpendingfailure of their firm. Second,the entities entering into some relationship with the firm, i.e.mostly the creditors of the firm,are interested in learning about a possible non-repayment of their receivablesin advance.

In the literature, the issue of corporate failure is denoted by different notions: corporate failure prediction, bankruptcy prediction, financial difficulty prediction, default prediction, credit risk assessment, early warning systems etc. Despite different terminology, the essence of the various papers is common – anticipating corporate insolvency. The reason is obvious – the insolvency is the underlying cause for extinction of business entities (Cisko and Klieštik, 2013).

Beaver (1966) was one of the first authors who successfully used financial ratios to address the problem of corporate failure prediction. Focusing only on selected simple financial ratios as predictors of corporate failure, however, was largely disputed. In order to overcome this deficiency, the models utilizing complex multidimensional statistical and data-mining methods have been theoretically and practically studied. The literature is dominated especially by statistical discriminant analysis (DA), econometric logistic regression (LR) and neural networks (NN). Statistical DA was firstly used to discriminate between bankrupt and non-bankrupt firms by Altman (1968). Although, the Altman´s model provided adequate results within sample, its ability to forecast out-of-sample proved to be deficient (see e.g. Grice and Ingram, 2001).As the next in line, LR was applied for the corporate failure prediction. The first author who introduced the use of LR was Ohlson (1980). Since that time, a noteworthy number of studies for LR application in bankruptcy classification followed e.g. (Zavgren, 1985).

In addition to theabovementioned methods, also the non-parametric DEAhas seen expanding array of its application in corporate failure assessment in recent years.It is critical to emphasize, that the use of DEA in the context of corporate failure prediction is different from the conventional application of DEA for efficiency analysis.To date, several researchers (Cielen et al.,2004;Ravikumar and Ravi, 2007;Sueyoshi, 2006;Premachandra et al. 2009, 2011) have used DEA in corporate failure assessment.

The main aim of this study is to utilize DEA within the framework of corporate failure predictionto develop a simple bankruptcy assessment tool. To this end, we propose amodification of the additive model of Charnes et al. (1985) for the prediction of corporate failure.

2. Methodology

Our analysis consists of the following four steps: variable selection, construction of the corporate failure frontier, computation of failure measures and assessment of prediction capability of the model.

2.1 Variable selection

In the first step we should identify financial ratios with satisfactory discriminant ability. This selection can be based on our domain knowledge or we can compare values of financial ratios between default and non-default firms using appropriate statistical methods, e.g. comparison of boxplots, independent samples t-test, Wilcoxon-Mann-Whitney test, bootstrap confidence interval for means and medians etc.

2.2 Construction of the corporate failure frontier

In the context of corporate failure assessment, Premachandra et al. (2009) proposed to construct the bankruptcy frontier in the following sense. Financial ratios are considered as inputs (outputs) if their small (large) values could possibly cause financial distress. All linear combinations of inputs and outputs form the Corporate Failure Possibility Set (CFPS). Then the firms with small values of inputs and large values of outputs are at risk of failure. Hence, this input-output classification will identify the Corporate Failure Frontier (CFF), and indicates those firms that are about to fail. In this way, the CFF is constructed (see Figure 1) instead of the Production Possibility Frontier(PPF) that is conventionally considered in DEA.

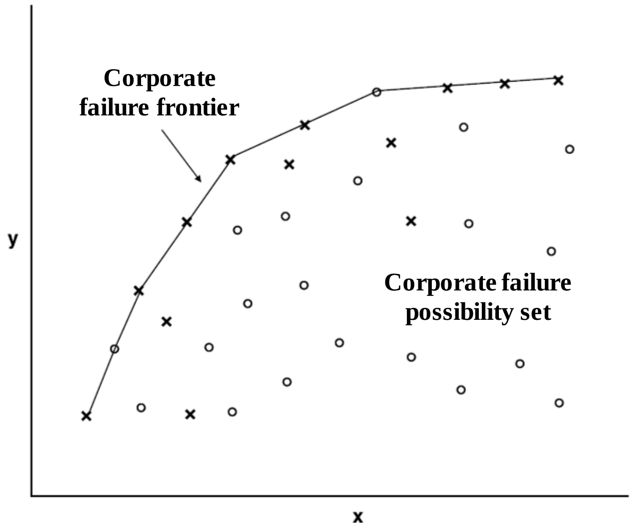

Figure 1. Corporate failure frontier and corporate failure possibility set for one input (x) and one output (y). The symbol (○) indicates a non-default firm and the symbol (×) indicates a default firm.
Source: Premachandra et al. (2009).

In this regard, it is critical to observe that the results of DEA are generally highly sensitive to outliers, i.e. the constructed CFF and therefore the classification of firms as default or non-default can be distorted by the presence of extreme values. This means thatifthedatasetis contaminated by the outliers, i.e. there are firms manifestly different from the others, DEAloses its ability toidentify otherthreatenedfirms. To overcome this limitation, we propose the following two-step procedure:

Step 1 We construct the CFF based on the full sample of n firms. This way we identify firms forming the CFF, i.e. firms which are about to fail. Let us denote the number ofthese firmsby k, $k \le n$.

Step 2 We omit one of firms that form the CFF in Step 1 and reconstruct the CFF. This step we repeat k-times, each time we omit another firm from the CFF constructed in Step 1. This partially eliminates negative influence of outliers on the results achieved.

This procedure,similarly as in the Altman´s model, results in the followingthreezones:

Distress zone: It contains firms which in Step 1form the CCF, i.e. firms at risk of failure.

Grey zone: It contains firms which in the Step 1 do not belong to the CCF and in Step 2formthe CCF atleast once, i.e. firms threatened by the financial distress.

Safe zone: It containsfirms that do not belong to the CCFin all previous $k+1$ steps, i.e. firms with good financial situation.

It should be noted that we can assume even more detailed classification of the groups depending on the number of occurrences at the CFF. This would result even to $k+2$ different groups.

2.3 Computation of failure measures

The financial ratios take oftenthe negative values. Although there exists a number of methods dealing with negative data, e.g. (Portela et al., 2004;Emrouznejad et al., 2010), we restrict ourselves to the additive model of Charnes et al. (1985).The main reason is its simplicity and the fact that is not necessary toselectthe input or outputorientationof the model. In this model the inefficienciesof inputs and outputs aresimultaneouslyincluded in evaluation. The additive model measuresefficiency of a particular firm $o, o \in \{1,\dots, n\}$ as follows:

$$\max_{\mathbf{s}^-,\mathbf{s}^+,\lambda} \ \mathbf{e}'\mathbf{s}^- + \mathbf{e}'\mathbf{s}^+ \qquad \text{subject to:} \qquad \begin{aligned} \mathbf{s}^- &= \mathbf{x}_o - \mathbf{X}\boldsymbol{\lambda}, \\ \mathbf{s}^+ &= \mathbf{Y}\boldsymbol{\lambda} - \mathbf{y}_o, \\ \boldsymbol{\lambda} \geq \mathbf{0}, &\ \mathbf{s}^- \geq \mathbf{0}, \mathbf{s}^+ \geq \mathbf{0}, \end{aligned} \tag{1}$$

where, n is the number of firms under consideration, m is the number of inputs, s is the number of outputs, $\mathbf{X}$ denotes a $m \times n$ matrix of inputs, $\mathbf{Y}$ denotes a $s \times n$ matrix of outputs, $\mathbf{e}'$ is a row vector with all elements equal to 1, $\mathbf{x}_o$ is a column vector of m inputs of the firm o, $\mathbf{y}_o$ is a column vector of s outputs of the firm o, $\mathbf{s}^-$ is a vector of m input slacks (excesses) of the firm o, $\mathbf{s}^+$ is a vector of s output slacks (shortfalls) of the firm $o$and $\boldsymbol{\lambda} \in R^n$ is an intensity variable vector connecting inputs and outputs.

Let $(\mathbf{s}^{-*}, \mathbf{s}^{-*}, \boldsymbol{\lambda}^*)$ be an optimal solution of (1). Then thefirm $o$formsthe CCF if and only if $\mathbf{s}^{-*} = \mathbf{0}$ and $\mathbf{s}^{+*} = \mathbf{0}$ .In the context of corporate failure assessment,the firmswith a high probability of their future failure tend to have a value for the objective function of the additive model (1) equal to zero, and the firmswith low probability oftheirfuturefailuretend to have these valuesgreater than zero.

2.4 Corporate failure assessment capability

We validate the corporate failure prediction capability of the proposed approachin the following way.Classify all n firms into the following six groups:

Group a	default firms in the Distress zone,
Group b	default firms in the Grey zone,
Group c	default firms in the Safe zone,
Group d	non-default firms in the Distress zone,
Group e	non-default firms in the Grey zone,
Group f	non-default firms in the Safe zone.

The firms belonging to Groups a, b, e and f are correctly classified, while the firms belonging to Groups c and d are classified incorrectly. According to Altman(1968, pp. 599), Group d is Type I error and Group c is Type II error.

Let n_i, $i = a,\dots,f$ denotes the number of firms belonging to the Group i. Evidently, it must hold that $\sum_{i=a}^{f} n_i = n$. Based onthese numberswe can define the following two indices:

$$I_{CC} = \frac{n_a + n_b + n_e + n_f}{n}, \tag{3}$$

$$I_{IC} = \frac{n_c + n_d}{n} = 1 - I_{CC}. \tag{4}$$

The misclassification rate is determined by $I_{IC} \in [0,1]$, while the correct classification rate is determined by $I_{CC} \in [0,1]$.

3. Population and sample

To illustrate the proposed model for corporate failure prediction, the primary data set on Slovak bankruptcies over the period 2009 – 2013wasobtained from the databasepurchased from CRIF – Slovak Credit Bureau, s.r.o,. The original data setconsists of more than 147 000firms from various sectors of economywith108 different financial indicatorsdrawn from the financial statements, i.e. balance sheets and income statements. To take into account the differences that may exist between different sectors within the economy, only one sector was selected. According to Statistical Classification of Economic Activities in the European Community, Rev. 2 (NACE Rev. 2), 14 563 firms belonging to Section G – Wholesale and retail trade; repair of motor vehicles and motorcycles, Division 46 – Wholesale trade, except of motor vehicles and motorcycles, were selected. A random subsample of 50 firms was drawn from these 14 563 firms for analysis. The subsample contained 12 default firms and 38 non-default firms.

3.1 Input and outputs variables considered in the analysis

Based on the Beaver´s(1966) assumptionthat thefinancial ratios are goodindicators ofthe financial distress of a firm, ten financial ratios (seven inputs and three outputs) were used in our analysis. Input variables were selected in such a waythat firms with smaller values for these variables are more likely to experience financial distress. Hence financial ratios used as inputs proxy for the financial strength and solvencyof firms. Input variables were represented by three liquidity ratios reflecting the firms´ ability to meet its obligations – acid ratio (AR), current ratio (CR) and working capital to total assets ratio (WCTA), one activity ratio reflecting how effectively the firm utilizes its resources – asset turnover (AT), one leveraging ratio expressing how the firm is sustainable and risky to lend future loans – equity ratio (ER) and two profitability ratios reflecting the firm´s ability to generate an acceptable rate of return – return on equity (ROE) and return on assets (ROA). The following formulas were used for the input variables computation:

- AR = (current assets – inventory) / current liabilities.
- CR = total current assets / total current liabilities.
- WCTA = working capital / total assets
- AT = total sales / total assets.
- ER = equity / total assets.
- ROE = EBIT / Equity.
- ROA = EBIT / Total Assets.

Output variables were selected in such a way that firms with higher values for these variables are more likely to experience financial distress. Henceforth financial ratios used as outputs proxy for the financial weakness and insolvency of firms. Output variables were represented by three debt ratios quantifying the firm's ability to repay long-term debt – debt ratio (DR), long-term debt ratio (LDR) and debt to equity ratio (DER). The following formulas were used to compute the output variables:

- DR = total liabilities / total assets.
- LDR = long-term debt / total assets.
- DER = total liabilities / equity.

All variables are computed at the end of the fiscal year immediately preceding the year of corporate failure.Summary statistics of all the input and output variables, computed separately for bankrupt and non-bankrupt firms, are presented in Table 1.

Table 1. Summary statistics for input and output variables (2009 – 2013)

Sample selection	Statistic	Input variables							Output variables		
		AR	CR	WCTA	AT	ER	ROE	ROA	DR	LDR	DER
Non-bankrupt firms	Mean	2.37	2.75	-1.36	5.55	-6.75	0.28	0.03	7.73	5.66	1.51
	Median	0.62	0.81	-0.15	1.23	0.22	0.16	0.01	0.77	0.00	1.22
	Standard deviation	9.11	9.34	5.02	26.03	37.15	0.76	0.60	37.15	34.47	7.43
Bankrupt firms	Mean	0.45	0.41	-22,264.6	0,27	- 48,271.91	46,18	-575,65	48,272.91	26,007.50	-1.77
	Median	0.53	0.31	-1.36	0.00	-0.85	0.01	-0.02	1.85	0.00	-1.10
	Standard deviation	0.40	0.43	51,980.03	0,46	112,717.9	158,86	1 278,14	112,717.9	60,740.43	21.36

Source: Author

Since most variables seemedto be skewed, we usedmedians for the purpose of comparison. For simplicity, we restricted ourselves to examining boxplots. They indicated that the median values for the bankrupt and non-bankrupt firms are different for all variables. These results suggest that the variables used in the analysis are appropriate for construction of classifiers.

4. Results and discussion

The additive models were computed using Josef Jablonský´s software DEA-Excel Solver 2014,that is a MS Excel based system for DEA models (https://webhosting.vse.cz/jablon/). Our results are summarized in Table 2.

Table 2. Summary of DEA results

	Distress zone	Grey zone	Safe zone	Total
No. of bankrupt firms	5	0	7	12
	41.7%	0%	58.3%	100%
No. of non-bankrupt firms	10	11	17	38
	26.3%	28.9%	44.7%	100%
$I_{CC} = 66\%$ $I_{IC} = 34\%$				

Source: Authors.

It is important to emphasize that the model based on data characterizing the companies of one country may not be successfully used to predict the corporate failure in other countries. For example, in the case of Altman model there is a significantly different informative value of the indicator of the market value of equity / book value of debt in the economy with developed capital market and in the economy with less developed capital market. For countries with significantly less developed capital market, which does not reflect market expectations, it is likely that this indicator will be for many firms biased. Moreover it should be further noted that differences may exist not only between countries but also between different sectors within the same country.

5. Conclusion and future extensions

In the presented paper we proposed a simply methodology how to utilize DEA as a bankruptcy assessment tool. Our approach was illustrated on a small sample of Slovak companies. In our opinion, it can be seen as valuable addition or complement to traditional models. In our future research we would like to further investigatethe prediction ability of DEA based prediction models and the possibility to combine them with other classifiers into meta-models with better prediction accuracy.

Acknowledgements

The support of the grant scheme VEGA, projects 1/0757/15 and 1/0647/14,is gladly acknowledged.

References

1. ALTMAN, E. I. 1968. Financial ratios, discriminant analysis and the prediction of corporate bankruptcy. In Journal of Finance, 1968, vol. 23, iss. 4, pp. 589-609.
2. BEAVER, W. H. 1966. Financial ratios as predictors of failure. In Journal of Accounting Research, 1966, vol. 4, supplement to iss. 4, pp. 71-111.
3. CHARNES, A. et al. 1985. Foundations of data envelopment analysis for Pareto Koopmans efficient empirical production functions. In Journal of Econometrics, 1985, vol. 30, iss. 1-2, pp. 91-107.
4. CIELEN, A., PEETERS, L., VANHOOF, K. 2004. Bankruptcy prediction using a data envelopment analysis. In European Journal of Operational Research, 2004, vol. 154, iss. 2, pp. 526-532.
5. CISKO, Š., KLIEŠTIK, T. 2013. Finančnýmanažmentpodniku II. Žilina : EDIS Publishing. 2013. 774 pp. ISBN 978-80-554-0684-8.
6. EMROUZNEJAD, A., ANOUZE, A., THANASSOULIS, E. 2010. A semi-oriented radial measure for measuring the efficiency of decision making units with negative data, using DEA. In European Journal of Operational Research, 2010, vol. 200, iss. 1, pp. 297-304.
7. GRICE, S., INGRAM, R. 2001. Tests of the generalizability of Altman's bankruptcy prediction model. In Journal of Business Research, 2001, vol. 54, iss. 1, pp.53-61.
8. OHLSON, J. 1980. Financial ratios and the probabilistic prediction of bankruptcy. In Journal of Accounting Research, 1980, vol. 18, iss. 1, pp. 109-131.
9. PORTELA, M. C. A. S., THANASSOULIS, E., SIMPSON, G. 2004. Negative data in DEA: A directional distance approach applied bank branches. In Journal of the Operational Research Society, 2004, vol. 55, iss. 10, pp. 1111-1121.
10. PREMACHANDRA, I. M. BHABRA, G. S., SUEYOSHI, T. 2009. DEA as a tool for bankruptcy assessment: a comparative study with logistic regression technique. In European Journal of Operational Research, 2009, vol. 193, iss. 2, pp. 412-24.
11. PREMACHANDRA, I. M., CHEN, Y., WATSON, J. 2011. DEA as a tool for predicting corporate failure and success: A case of bankruptcy assessment. In Omega, 2011, vol. 39, iss. 6, pp. 620-626.

12. RAVIKUMAR, P., RAVI, V. 2007. Bankruptcy prediction in banks and firms via statistical and intelligent techniques: A review. In European Journal of Operational Research, 2007, vol. 180, iss. 1, pp. 1-28.
13. SUEYOSHI, T. 2006. DEA-discriminant analysis: Methodological comparison among eight discriminant analysis approaches. In European Journal of Operational Research, 2006, vol. 169, iss. 1, pp. 47-272.
14. WARNER, J. 1977. Bankruptcy costs: Some evidence. In Journal of Finance, 1977, vol. 32, iss. 2, pp. 337-347.
15. ZAVGREN, CH. V. 1985. Assessing the vulnerability to failure of American industrial firms: A logistic analysis. In Journal of Business Finance & Accounting, 1985, vol. 12, iss. 1, pp. 19-45.